MTH 406: Differential geometry of curves and surfaces

(Due 13/03)

Homework IV: Orientation

Problems for turning in

1. Prove the assertions in 2.4(viii), and 2.4(xiii) - (xiv) of the Lesson Plan.

2. Show that a continuous unit normal field on a regular surface must be smooth.

3. Show that the real projective plane $\mathbb{R}P^2$ obtained by identifying the antipodal points in S^2 is a nonorientable regular surface.

4. Let $f : S \to S'$ be a diffeomrophism between regular surfaces.

   (a) Show that an orientation on S induces a naturally well-defined orientation on S' with respect to which f is orientation-preserving.

   (b) Show that an oreintable surface cannot be diffeomorphic to a nonorientable surface.

Problems for practice

1. Prove the assertions in 2.4(vii)(b)-(e) of the Lesson Plan.

2. Let S be a regular orientable surface with an orientation N. Show that if f is a rigid motion of $\mathbb{R}^3$, then $f(S)$ is a regular orientable surface with a naturally induced orientation N' given by $N'(f(p)) = df_p(N)$, for all $p \in S$.

3. Let S be a regular surface, $p \in S$, $v \in T_p(S)$ a nonzero vector, and N a unit normal vector to S at p. Prove that there exists a neighborhood $V \ni p$ in S the intersection of V with the plane $p + \text{span}\{v, N\}$ is the image of a regular curve.

4. Let S be a regular surface, and let $f : S \to \mathbb{R}$ be a smooth function. Show that there exists a unique tangent field ∇f on S (called the *gradient* of f) with the property

$$df_p(v) = \langle (\nabla f)(p), v \rangle, \ \forall p \in S \text{ and } v \in T_p(S).$$